IFRS 17 & Solvency II Workshop

Quantitative aspects of Solvency II

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Agenda

Monday, 15 July

- Recap of IFRS 17 Background
- General Measurement Model
- Reinsurance Held and Contracts Acquired
- Implementing IFRS 17

Tuesday, 16 July

- Measurement of direct participation contracts
- Illustrative examples of the Premium Allocation Approach
- Presentation of IFRS 17 Results
- Data management and calculation engines
- Background and scope of Solvency II
- Quantitative aspects of Solvency II

Wednesday 17 July

- Quantitative aspects of Solvency II (cont'd)
- Governance under Solvency II
- The Risk Management & Reporting Processes

The Standard Formula

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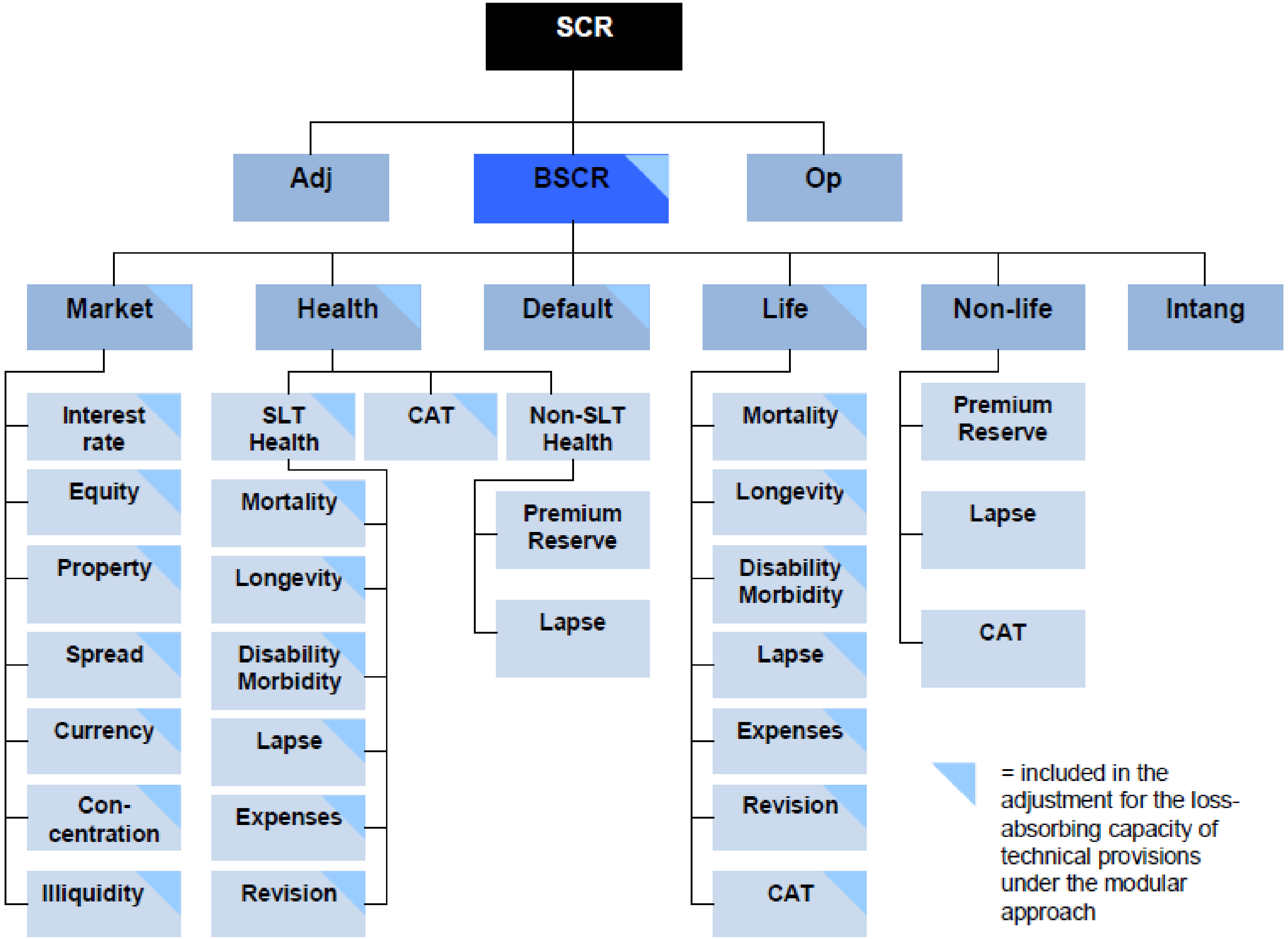
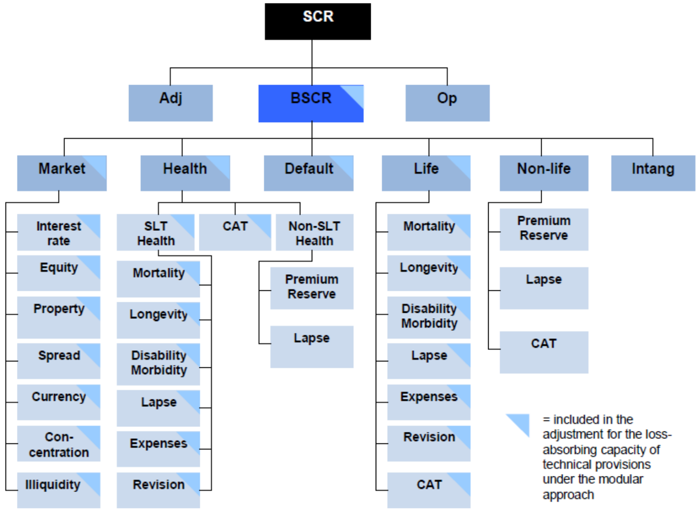
Overall structure of the solvency capital requirement

ACCORDING TO THE STANDARD FORMULA



Modules and sub-modules

- For each module and sub-module, the specifications are split into the following:
  - **description**: scope of module and definition of the relevant sub-risk
  - **input**: list of the input data requirements
  - **output**: output data generated by the module
  - **calculation**: how the output is derived from the input
  - **simplification**: how the calculation can be simplified under certain conditions
- There are three mutually exclusive underwriting modules:
  - life
  - health
  - non-life



Overall SCR calculation

Description	The SCR is the end result of the standard formula calculation
Input	$BSCR$ – basic solvency capital requirement SCR_{op} – capital requirement for operational risk Adj – adjustment for the risk absorbing effect of technical provisions and deferred taxes
Output	This module delivers the overall standard formula capital requirement
Calculation	$SCR = BSCR + Adj + SCR_{op}$

BSCR calculation

Description	The $BSCR$ is the solvency capital requirement before any adjustments, combiing capital requirements for six major risk categories
Input	SCR_{mkt} – capital requirement for market risk SCR_{def} – capital requirement for counterparty default risk SCR_{life} – capital requirement for life underwriting risk SCR_{nl} – capital requirement for non-life underwriting risk SCR_{health} – capital requirement for health underwriting risk $SCR_{intangibles}$ – capital requirement for intangible assets risk
Output	This module delivers $BSCR$, the basic solvency capital requirement
Calculation	$BSCR = \sqrt{\sum_{n} Corr_{ij} \times SCR_i \times SCR_j} + SCR_{intangibles}$ where $Corr_{ij}$ are the entries of the correlation matrix, and SCR_i, SCR_j are the capital requirements for the individual risks according to the rows and columns of the correlation matrix

Correlation matrix for the BSCR calculation

- The standard formula assumes linear correlations among the risks as follows:

i, j	Market	Default	Life	Health	Non-life
Market	1.00				
Default	0.25	1.00			
Life	0.25	0.25	1.00		
Health	0.25	0.25	0.25	1.00	
Non-life	0.25	0.50	0.00	0.00	1.00

BSCR calculation

Description	The $BSCR$ is the solvency capital requirement before any adjustments, combiing capital requirements for six major risk categories
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Non-life underwriting risk

Description	Risk arising from non-life insurance obligations, in relation to the perils covered and the processes used in the conduct of business
Input	NL_{pr} – capital requirement for non-life premium and reserve risk NL_{lapse} – capital requirement for non-life lapse risk NL_{CAT} – capital requirement for non-life catastrophe risk
Output	SCR_{nl}, the capital requirement for non-life underwriting risk
Calculation	$SCR_{nl} = \sqrt{\sum CorrNL_{r,c} \times NL_r \times NL_c}$ where $CorrNL_{r,c}$ are the entries of the correlation matrix, andNL_r, NL_c are the capital requirements for the individual risks according to the rows and columns of the correlation matrix

Correlation matrix for the non-life underwriting module

- The non-life underwriting module assumes linear correlations among the risks as follows:

$CorrNL$	NL_{pr}	NL_{lapse}	NL_{CAT}
NL_{pr}	1.00		
NL_{lapse}	0.00	1.00	
NL_{CAT}	0.25	0.00	1.00

Non-life premium and reserve submodule

Description	Combination of the two main sources of non-life underwriting risk: premium risk and reserve risk
Input	PCO_s – best estimate for claims outstanding for each segment $s = 1, \dots, 12$ P_s – estimate of premiums of segment s to be earned during the following 12 months $P_{(last,s)}$ – premiums of segment s earned during the previous 12 months $FP_{(existing,s)}$ – expected present value of earned premiums of segment s after the following 12 months $FP_{(future,s)}$ – expected present value of earned premiums of segment s where initial recognition is within 12 months but excluding premiums to be earned during the 12 months after recognition
Output	NL_{pr}, the capital requirement for premium and reserve risk
Calculation	$NL_{pr} = \left[\frac{exp\left(\Phi^{-1}(99.5\%) \sqrt{\ln(1 + c^{prem,res^2})} \right)}{\sqrt{1 + c^{prem,res^2}}} - 1 \right] V \approx 3 \cdot \sigma \cdot V$ where σ is the standard deviation for premium and reserve risk considering all segments and V is a volume measure determined with the input premiums and claims for all segments. A correlation matrix between segments and standard deviations per segment (gross and net of reinsurance) are prescribed.

Quantitative risk models

1- Notional amount approach

Features	• Oldest methodology, where risk is the weighted average of notional amounts and risk factors • Still widely used – Basel Accords – Certain modules and sub-modules of standard formula of Solvency II
Advantages	• By far, the simplest approach
Disadvantages	• No distinction made between short and long positions • No account for diversification effects • Notional amounts may not necessarily represent economic values

2- Risk measures based on loss distributions

Features	• Modern risk measures are characteristics of the underlying loss distribution over some predetermined time horizon • Examples: variance (or sd), VaR, TVaR
Advantages	• The notion of loss distributions is intuitively appealing • If estimated properly, loss distributions reflect diversification effects
Disadvantages	• Estimates of loss distributions are based on historical data • It is difficult to estimate loss distributions accurately • Risk dependencies are generally not well understood

3- Scenario-based risk measures

Features	• Used widely for stress testing • Consideration of possible future risk factor changes (e.g., a 25% drop in the stock market) • The risk of a portfolio is the maximum weighted loss under all scenarios: If $X = \{x_1, x_2, \dots, x_n\}$ denotes the risk-factor changes (i.e., scenarios) with weights $\mathrm{w} = \{w_1, w_2, \dots, w_n\}$, then the risk is expressed as $\psi_{X,w} = \max_{1 \le i \le n} \{w_i\, L(x_i)\}$ where $L(x)$ is the loss of the portfolio if the scenario x occurs.
Advantages	• Useful for portfolios with a small number of risk factors • Good complement to risk measures based on loss distributions
Disadvantages	• Difficult to determine scenarios and weights

VaR

$\text{VaR}_\alpha[L] = \inf\{x \in \mathbb{R}\colon F_L(x) \geq \alpha\}$, where L is a loss random variable with distribution function F and $\alpha \in (0,1)$

- VaR was popularized in 1994 by JP Morgan, *RiskMetrics* ("The Weatherstone 4:15 Report") and is by far the most widely used metric
- Examples:

Distribution of L	Value-at-Risk
$N(\mu, \sigma^2)$	$\mu + \sigma\Phi^{-1}(\alpha)$
$t_v(\mu, \sigma^2)$	$\mu + \sigma t_v^{-1}(\alpha)$
Exponential $(\text{scale} = \theta)$	$-\theta \ln(1-\alpha)$
Pareto II $(\text{scale} = \theta, \text{shape} = \kappa)$	$\theta[(1-\alpha)^{-1/\kappa} - 1]$

TVaR

$\mathrm{TVaR}_\alpha[L] = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_\mathrm{u}(L)du$, where L is a loss rv with distribution function F, $\mathrm{E}[L] < \infty$ and $\alpha \in (0,1)$

- TVaR_α is the average VaR_u for all $\geq \alpha$, and is the second most widely used metric in practice
- TVaR_α looks further into the tail of the distribution
- Examples:

Distribution of L	Tail Value at Risk
$N(\mu, \sigma^2)$	$\mu + \sigma \frac{\varphi\left(\Phi^{-1}(\alpha)\right)}{1-\alpha}$
$t_\upsilon(\mu, \sigma^2), \upsilon > 1$	$\mu + \sigma f_{t_\upsilon}(t_\upsilon^{-1}(\alpha)) \frac{(\upsilon + t_\upsilon^{-1}(\alpha)^2)}{(1-\alpha)(\upsilon - 1)}$
Exponential $(\mathrm{scale} = \theta)$	$-\theta \ln(1-\alpha) + \theta$
Pareto II $(\mathrm{scale} = \theta, \mathrm{shape} \ = \kappa)$	$\theta\left[(1-\alpha)^{-1/\kappa} - 1\right] + \frac{\theta(1-\alpha)^{-1/\kappa}}{\kappa - 1}$

Axioms of coherence

BACKGROUND

- Paper by Artzner et al (1999)
- We will assume that risk measures ρ are defined on a linear space of random variables $\mathcal{M}$ (including constants), that is, $\rho: \mathcal{M} \to \mathbb{R}$
- Let be L a loss random variable

Axioms of coherence

MONOTONICITY

$$L_1, L_2 \in \mathcal{M}, L_1 \leq L_2 \Rightarrow \rho(L_1) \leq \rho(L_2)$$

- Positions which lead to a higher loss in every state of the world, require more capital

Axioms of coherence

TRANSLATION INVARIANCE

$$\rho(L+l) = \rho(L) + l, \;\; L \epsilon \mathcal{M} \text{ and } l \in \mathbb{R}$$

- By adding l to a position with a loss L, we alter the capital requirement accordingly
- Alternatively, if $\rho(L) > 0$ and $l = -\rho(L)$, then $\rho\big(L - \rho(L)\big) = \rho(L+l) = \rho(L) + l$, so that adding $\rho(L)$ to a position with losss L makes it acceptable

Axioms of coherence

SUBADDITIVITY

$$\rho(L_1 + L_2) \leq \rho(L_1) + \rho(L_2), \quad L_1, L_2 \,\epsilon\, \mathcal{M} \text{ and } l \in \mathbb{R}$$

- Reflects the idea of diversification
- VaR may be superadditive under some circumstances
  - The random variables have skewed distributions
  - The random variables have special dependence (for example, functional dependence)
  - The random variables have heavy-tailed distributions

Axioms of coherence

POSITIVE HOMOGENEITY

$$\rho(\lambda L) = \lambda\rho(L), \quad L \in \mathcal{M} \text{ and } \lambda > 0$$

- Notice that if λ is large, then liquidity concerns arise, which leads to the concept of convex risk measures

Coherent risk measures

DEFINITION

- A coherent risk measure is a risk measure ρ that satisfies
  - monotonicity
  - translation invariance
  - subadditivity
  - positive homogeneity

- Subadditivity and positive homogeneity together can be thought of "convexity"

$$\rho(\lambda L_1 + (1-\lambda)L_2) \leq \lambda\rho(L_1) + (1-\lambda)\rho(L_2), \quad L_1\,, L_2 \;\epsilon\; \mathcal{M} \text{ and } \lambda \in [0:1]$$

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The SCR under Solvency II

- Defined as the amount of capital that enables the insurer to meet its obligations over one year ($\Delta t = 1$) with probability $\alpha = 0.995$ (that is, 199 out of 200 times)

Let V_t denote the capital. The insurance company determines the minimum amount of extra capital x_o to stay solvent at Δt with probability of at least $\alpha = 0.995$. Therefore:

$$\begin{aligned} x_0 &= \inf\{x \in \mathbb{R} : \Pr[V_{t+1} + x(1+r) \geq 0] \geq \alpha\} \\ &= \inf\left\{x \in \mathbb{R} : \Pr\left[-\left(\frac{V_{t+1}}{1+r} - V_t\right) \leq x + V_t\right] \geq \alpha\right\} \\ &= \inf\{x \in \mathbb{R} : \Pr[L_{t+1} \leq x + V_t] \geq \alpha\} \\ &= \inf\{x \in \mathbb{R} : F(x + V_t) \geq \alpha\} \\ &= \mathrm{VaR}_\alpha[L_{t+1}] - V_t \end{aligned}$$

Therefore, $SCR = V_t + x_0 = \mathrm{VaR}_\alpha[L_{t+1}]$

The SCR is the sum of the capital available today and the capital required to stay solvent in Δt with 99.5% probability. If $x_0 < 0$, then the company has already enough capital.

Illustrative examples of a market risk model

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Model 1—the delta-gamma model

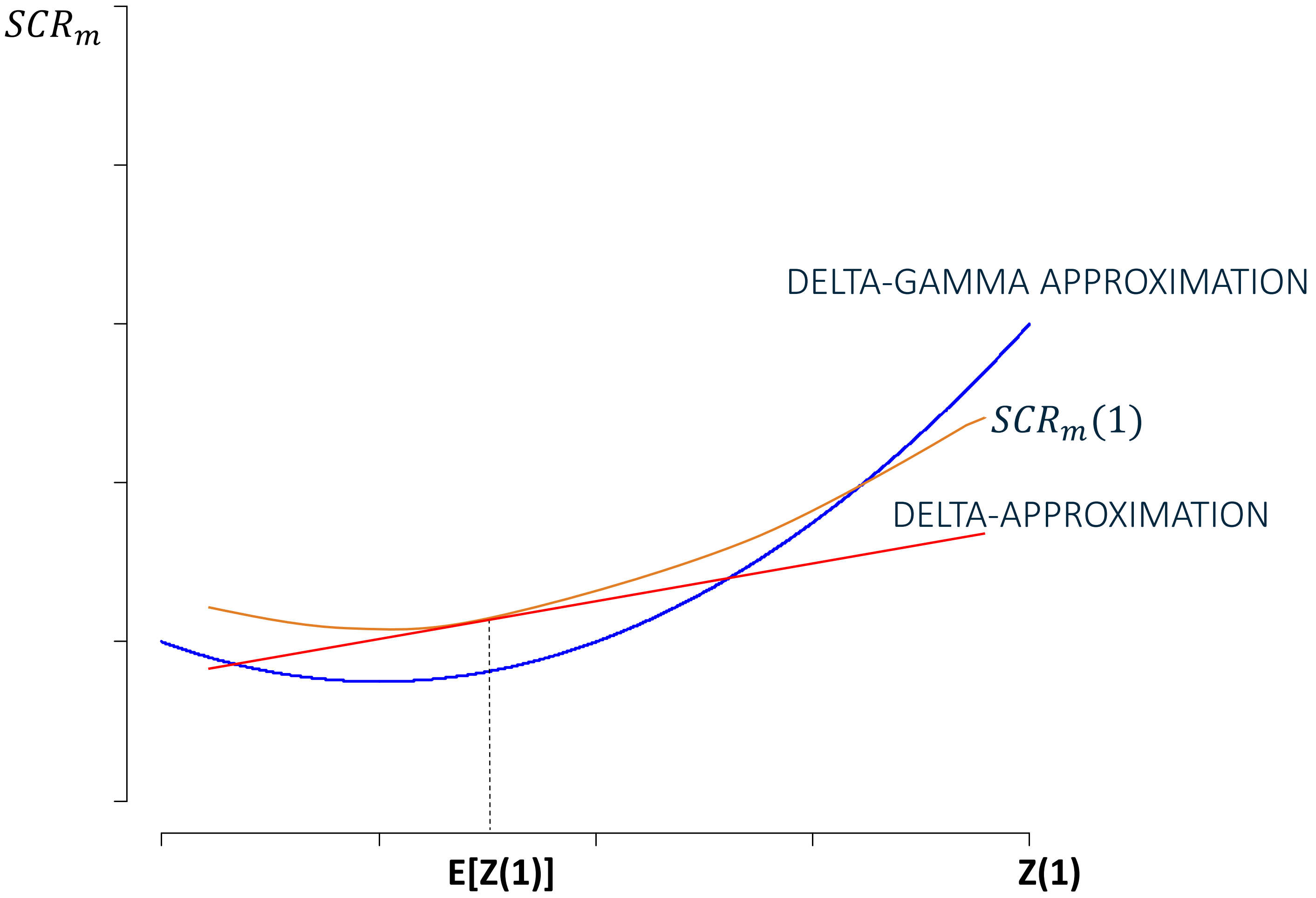
Suitable approximation of the required capital for certain market risks and mortality risks

- Let SCR_m be the capital requirement for market risk
- SCR_m is a function of random risk factors $\boldsymbol{Z}_i(1)$
- $SCR_m = SCR_m\big(t; \boldsymbol{Z}(t)\big) = SCR_m\big(t;\ Z_1(t), \dots, Z_d(t)\big)$
- $\boldsymbol{Z}(t) = \big(Z_1(t), Z_2(t), \dots, Z_d(t)\big)^T$
- Using the first two terms of the Taylor series:

$$SCR_m(\boldsymbol{Z}(1)) \risingdotseq SCR_m\big(\boldsymbol{Z}(0)\big) + \sum_{i=1}^{d} \frac{\partial SCR_m\big(\boldsymbol{Z}(0)\big)}{\partial \boldsymbol{z}_i} X_i + \frac{1}{2} \sum_{i=1}^{d} \sum_{k=1}^{d} \frac{\partial^2 SCR_m\big(\boldsymbol{Z}(0)\big)}{\partial \boldsymbol{z}_i \partial \boldsymbol{z}_k} X_i X_k$$

$$\Delta SCR_m(1) \risingdotseq \sum_{i=1}^{d} \frac{\partial SCR_m\big(\boldsymbol{Z}(0)\big)}{\partial \boldsymbol{z}_i} X_i + \frac{1}{2} \sum_{i=1}^{d} \sum_{k=1}^{d} \frac{\partial^2 SCR_m\big(\boldsymbol{Z}(0)\big)}{\partial \boldsymbol{z}_i \partial \boldsymbol{z}_k} X_i X_k = \boldsymbol{\delta}^T \mathbf{X} + \frac{1}{2} \mathbf{X}^T \boldsymbol{\Gamma} \mathbf{X}$$

Delta-gamma approximation



The random variable $\delta^T \mathbf{X}$ + ½ $\mathbf{X}^T \Gamma$ $\mathbf{X}$

- Through the “delta—gamma” we obtain a rv that models the underlying risk
- $\Delta SCR_m(1) \fallingdotseq \boldsymbol{\delta}^T \mathbf{X} + \frac{1}{2} \mathbf{X}^T \boldsymbol{\Gamma} \mathbf{X}$
- $\mathbf{X} = \mathbf{X}(1) = \left(X_1(1), X_2(1), \dots, X_d(1)\right)^T$
- $\boldsymbol{\delta} = (\delta_1, \delta_2, \dots, \delta_d)^T$
- $\delta_i = \frac{\partial SCR_m(\mathbf{Z}(0))}{\partial z_i}$
- $\Gamma_{ik} = \frac{\partial^2 SCR_m(\mathbf{Z}(0))}{\partial z_i \partial z_k}$
- It is assumed that the risk factors follow a Normal distribution, therefore $\mathbf{X}$ is also Normal
- We obtain random vectors that follow a multivariate Normal distribution

Estimation of the sensitivities

Sensitivity	Estimator
$\delta_i = \frac{\partial SCR_m(\mathbf{Z}(0))}{\partial z_i}$	$\frac{SCR_m(\dots, z_i + h_i, \dots) - SCR_m(\dots, z_i - h_i, \dots)}{2h_i}$
$\Gamma_{ii} = \frac{\partial^2 SCR_m(\mathbf{Z}(0))}{\partial z_i^2}$	$\frac{SCR_m(\dots, z_i + h_i, \dots) - SCR_m(\dots, z_i, \dots)}{h_i^2} + \frac{SCR_m(\dots, z_i - h_i, \dots) - SCR_m(\dots, z_i, \dots)}{h_i^2}$
$\Gamma_{ik} = \frac{\partial^2 SCR_m(\mathbf{Z}(0))}{\partial z_i \partial z_k}$ $(i \neq k)$	$\frac{SCR_m(\dots, z_i + h_i, \dots, z_k + h_k, \dots) - SCR_m(\dots, z_i + h_i, \dots, z_k - h_k, \dots)}{4h_i h_k}$ $+ \frac{SCR_m(\dots, z_i - h_i, \dots, z_k - h_k, \dots) - SCR_m(\dots, z_i - h_i, \dots, z_k + h_k, \dots)}{4h_i h_k}$

Illustrative example

- Let's consider an endowment policy
  - Sum insured is USD 1m, term is 10 years
  - Portfolio of 100 insureds age 40
- Deterministic mortality assumptions, no lapses
- Spot rates: risk-free yield curve denominated in USD
- Invested assets denominated in USD, with market value of USD 105m
  - basket of stocks
  - zero-coupon bonds of 1—10 year terms

Implementation

- 12 risk factors
  - 10 spot rates (1—10 years)
  - 1 stock index factor
  - 1 mortality factor
- $\boldsymbol{h}$ is the vector of changes in risk factors
- Let $\boldsymbol{h} = (0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.10, 0.10)^T$
- Let's use additive factors for the changes in spot rates, and multiplicative factors for the changes of stock prices and mortality rates (this is arbitrary!)

Model output

From $\Delta SCR_m(1) \doteqdot \sum_{i=1}^{d} \frac{\partial SCR_m(\boldsymbol{Z}(0))}{\partial \boldsymbol{z}_i} X_i + \frac{1}{2} \sum_{i=1}^{d} \sum_{k=1}^{d} \frac{\partial^2 SCR_m(\boldsymbol{Z}(0))}{\partial \boldsymbol{z}_i \partial \boldsymbol{z}_k} X_i X_k = \boldsymbol{\delta}^T \mathbf{X} + \frac{1}{2} \mathbf{X}^T \boldsymbol{\Gamma} \mathbf{X}$

we generate $n = 10^5$ realizations of a multivariate normal distribution, with the appropriate volatilities and correlation matrix.

The **R** implementation of the model yields:

```
> model_output
                                              Monetary amounts in USD m
Net cash flows at t=0:                                          18.684
Expected net cash flows at t=1:                                 19.919
Risk capital for mortality risk:                                 0.140
Risk capital for market risk:                                   24.002
Diversification benefit:                                        -1.773
Solvency capital requirement (SCR):                             22.369
Market value of assets (MVA):                                  105.000
SCR as % of MVA:                                                21.304
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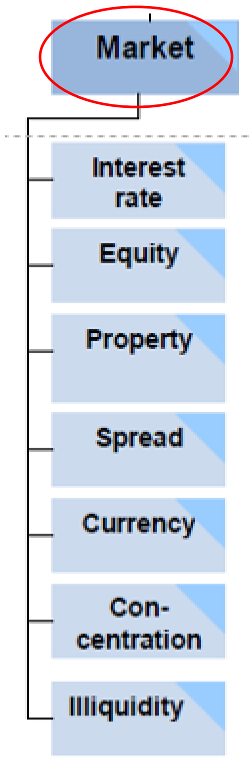
Model 2—Market risk under the Standard Formula



DEFINITION

Risks that arise from adverse changes to a company's financial position due to changes of market prices of financial instruments

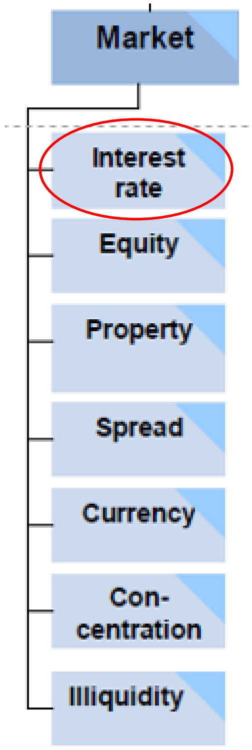
METHODOLOGY

- Modular approach
- Linear correlations used for the aggregation of individual components

$$SCR_m = \sqrt{\sum_i \sum_j Corr_{ij} \, SCR_i \, SCR_j}$$

$Corr$	Interest rate	Equity	Property	Spread	Currency	Concentration
Interest rate	1	0 | (0.5)	0 | (0.5)	0 | (0.5)	0	0.25
Equity	0 | (0.5)	1	0.75	0.75	0	0.25
Property	0 | (0.5)	0.75	1	1	0	0.25
Spread	0 | (0.5)	0.75	0.5	1	1	0
Currency	0	0	0	1	1	0
Concentration	0.25	0.25	0.25	0	0	1

Interest rate sub-module



DEFINITION

Risks that arise from adverse changes to a company's financial position due to changes in interest rate curves

METHODOLOGY

- Prescribed instantaneous shock factors to interest rates
- The risk is measured as impact on net value of assets when rates go up or down, including adjusments for future discretionary bonuses

$$\mathrm{SCR} = \mathrm{BSCR} + \mathrm{Adjustment} = \mathrm{BSCR} - \min(\mathrm{BSCR} - \mathrm{BSCR}^a, \mathrm{FDB})$$

$$\mathrm{BSCR} = \begin{cases} \Delta\mathrm{NAV}_{up} & \text{if } \max\left(\Delta\mathrm{NAV}^a_{up}, \Delta\mathrm{NAV}^a_{down}\right) = \Delta\mathrm{NAV}^a_{up} \\ \Delta\mathrm{NAV}_{down} & \text{if } \max\left(\Delta\mathrm{NAV}^a_{up}, \Delta\mathrm{NAV}^a_{down}\right) = \Delta\mathrm{NAV}^a_{down} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{BSCR}^a = \max\left(\Delta\mathrm{NAV}^a_{up}, \Delta\mathrm{NAV}^a_{down}\right)$$

Term	Incr.	Decr.
1	70%	75%
2	70%	65%
3	64%	56%
4	59%	50%
5	55%	46%
6	52%	42%
7	49%	39%
8	47%	36%
9	44%	33%
10	42%	31%
11	39%	30%
12	37%	29%
13	35%	28%
14	34%	28%
15	33%	27%
::::	::::	::::

Illustrative example

- A company issues an investment contract with an annual discretionary bonus
  - no mortality risk
  - assets are invested at the risk-free rate
  - liabilities are replicated using risk-free bonds
- Cash flow estimates are:

year	E[risk.free]	shocked.rate	gtd.rate	zero.bonds	pv.assets	repl.cf	stat.reserve	bonus	CF.ben	pv.liab		
0												
1	0.243%	0.243%	0.5%	1,500	4,068	1,000	3,938	0	1,000	3,938		scenario
2	0.671%	0.671%	0.5%	1,000	2,571	900	2,958	3	903	2,941		BE
3	1.095%	1.095%	0.5%	500	1,580	600	2,072	10	610	2,046		
4	1.428%	1.428%	0.5%	500	1,090	500	1,483	12	512	1,448		mitigating effect
5	1.684%	1.684%	0.5%	250	607	400	990	10	410	953		n/a
6	1.880%	1.880%	0.5%	250	369	300	595	7	307	564		
7	2.020%	2.020%	0.5%	100	136	200	298	4	204	277		**NAV**
8	2.113%	2.113%	0.5%	50	45	100	100	1	101	91		**129**

Changes in net asset values ($\Delta \mathrm{NAV}_{up}, \Delta \mathrm{NAV}_{down}$)

year	E[risk.free]	shocked.rate	gtd.rate	zero.bonds	pv.assets	repl.cf	stat.reserve	bonus	CF.ben	pv.liab		
0												
1	0.243%	0.413%	0.5%	1,500	4,019	1,000	3,938	0	1,000	3,875		scenario
2	0.671%	1.141%	0.5%	1,000	2,525	900	2,958	3	903	2,879		UP
3	1.095%	1.796%	0.5%	500	1,540	600	2,072	10	610	1,990		
4	1.428%	2.271%	0.5%	500	1,057	500	1,483	12	512	1,400		mitigating effect
5	1.684%	2.610%	0.5%	250	584	400	990	10	410	916		OFF
6	1.880%	2.858%	0.5%	250	353	300	595	7	307	538		
7	2.020%	3.010%	0.5%	100	129	200	298	4	204	263		**NAV**
8	2.113%	3.106%	0.5%	50	42	100	100	1	101	86		**143**

year	E[risk.free]	shocked.rate	gtd.rate	zero.bonds	pv.assets	repl.cf	stat.reserve	bonus	CF.ben	pv.liab		
0												
1	0.243%	0.000%	0.5%	1,500	4,134	1,000	3,938	0	1,000	4,022		scenario
2	0.671%	0.000%	0.5%	1,000	2,634	900	2,958	3	903	3,022		DOWN
3	1.095%	0.095%	0.5%	500	1,634	600	2,072	10	610	2,119		
4	1.428%	0.428%	0.5%	500	1,134	500	1,483	12	512	1,510		mitigating effect
5	1.684%	0.684%	0.5%	250	637	400	990	10	410	1,001		OFF
6	1.880%	0.880%	0.5%	250	390	300	595	7	307	596		
7	2.020%	1.020%	0.5%	100	145	200	298	4	204	295		**NAV**
8	2.113%	1.113%	0.5%	50	48	100	100	1	101	97		**111**

Changes in adjusted net asset values ($\Delta \text{NAV}_{up}^{a}, \Delta \text{NAV}_{down}^{a}$)

year	E[risk.free]	shocked.rate	gtd.rate	zero.bonds	pv.assets	repl.cf	stat.reserve	bonus	CF.ben	pv.liab		
0												
1	0.243%	0.413%	0.5%	1,500	4,019	1,000	3,938	0	1,000	3,926		scenario
2	0.671%	1.141%	0.5%	1,000	2,525	900	2,958	16	916	2,930		UP
3	1.095%	1.796%	0.5%	500	1,540	600	2,072	23	623	2,029		
4	1.428%	2.271%	0.5%	500	1,057	500	1,483	23	523	1,426		mitigating effect
5	1.684%	2.610%	0.5%	250	584	400	990	18	418	931		ON
6	1.880%	2.858%	0.5%	250	353	300	595	12	312	546		
7	2.020%	3.010%	0.5%	100	129	200	298	7	207	266		**NAV**
8	2.113%	3.106%	0.5%	50	42	100	100	2	102	86		**92**

year	E[risk.free]	shocked.rate	gtd.rate	zero.bonds	pv.assets	repl.cf	stat.reserve	bonus	CF.ben	pv.liab		
0												
1	0.243%	0.000%	0.5%	1,500	4,134	1,000	3,938	0	1,000	3,980		scenario
2	0.671%	0.000%	0.5%	1,000	2,634	900	2,958	0	900	2,980		DOWN
3	1.095%	0.095%	0.5%	500	1,634	600	2,072	0	600	2,080		
4	1.428%	0.428%	0.5%	500	1,134	500	1,483	0	500	1,481		mitigating effect
5	1.684%	0.684%	0.5%	250	637	400	990	1	401	983		ON
6	1.880%	0.880%	0.5%	250	390	300	595	2	302	587		
7	2.020%	1.020%	0.5%	100	145	200	298	1	201	291		**NAV**
8	2.113%	1.113%	0.5%	50	48	100	100	0	100	96		**154**

Calculations

From the tables, we populate the formulas:

$$\Delta\text{NAV}_{up} = -\min\left(0, \text{NAV}_{up} - \text{NAV}_{be}\right) = -\min(0, 143 - 129) = 0$$

$$\Delta\text{NAV}_{down} = -\min(0, \text{NAV}_{down} - \text{NAV}_{be}) = -\min(0, 111 - 129) = 18$$

$$\Delta\text{NAV}_{up}^{a} = -\min\left(0, \text{NAV}_{up}^{a} - \text{NAV}_{be}\right) = -\min(0, 92 - 129) = 37$$

$$\Delta\text{NAV}_{down}^{a} = -\min(0, \text{NAV}_{down}^{a} - \text{NAV}_{be}) = -\min(0, 154 - 129) = 0$$

$$\text{BSRC} = 0$$

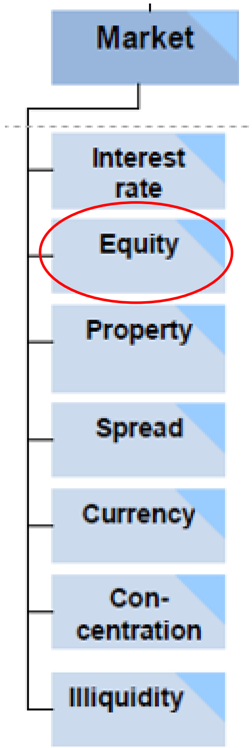
$$\text{BSRC}^{a} = 37$$

$$\text{FDB} = 45$$

$$\text{SCR} = \text{BSCR} - \min(\text{BSCR} - \text{BSCR}^{a}, \text{FDB}) = 0 - \min(0 - 37, 35) = 37$$

SOCIETY OF ACTUARIES®

Equity risk sub-module



DEFINITION

Risk of changes in a company's financial position due to changes in market prices of equity investments

METHODOLOGY

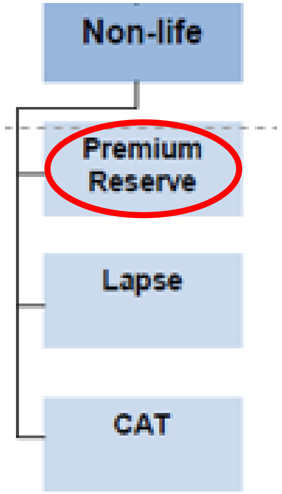
- Equities are classified into two types
- Type I—shares listed in regulated markets of EEA or OECD member countries
- Type II—shares listed only in emerging markets, non-listed shares, hedge funds and instruments not otherwise included in any other category of the market risk module
- Shocks of 46.5% and 56.5% for types I and II, respectively, plus miscellaneous adjustments
- Aggregation is achieved through:

$$SCR_{acc} = \sqrt{SCR_1^2 + 2(0.75)SCR_1\,SCR_2 + SCR_2^2}$$

Illustrative example of a non-life risk model

SOCIETY OF ACTUARIES®

Premium and reserve risk



DEFINITION

Risks that arise from:

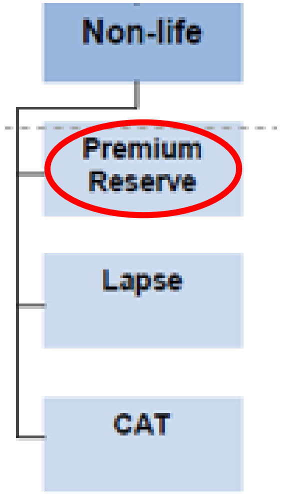
- insufficient premiums to cover obligations to policyholders
- larger-than-expected number of claims
- larger-than-expected amount of claims
- insufficient reserves to cover obligations to policyholders

METHODOLOGY

The combined premium and reserve risk follows a log-normal distribution

$$SCR_{NL}^{Prem,Res} = \mu \left(\frac{exp\left(\Phi^{-1}(99.5\%)\sqrt{\ln(1 + c^{prem,res^2})}\right)}{\sqrt{1 + c^{prem,res^2}}} - 1 \right)$$

Premium and reserve risk



DEVELOPMENT

Premium risk: $R_{NL}^{Prem} = \text{claims} + \text{expenses} - \text{P}$

Reserve risk: $R_{NL}^{Res} = \text{claims} + \text{expenses} + \text{reserve}^{final} - \text{reserve}^{initial}$

Estimation: $\widehat{R_{NL}^{Prem,Res}} = R_{NL}^{Prem} + \text{P} + R_{NL}^{Res} + \text{reserve}^{initial}$

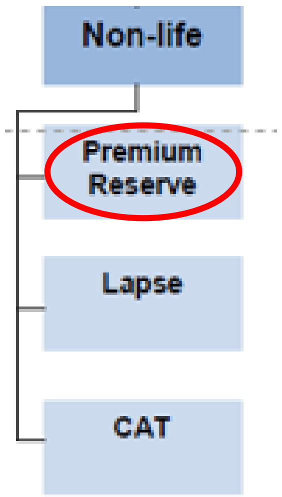
Modeling: $\ln(\widehat{R_{NL}^{Prem,Res}}) \sim \mathcal{N}\left(\ln\mu - \frac{1}{2}\ln\left(1 + c^{prem,res^2}\right), \ln(1 + c^{prem,res^2})\right)$

$$SCR_{NL}^{Prem,Res} = \text{VaR}_{99.5\%}\left(\widehat{R_{NL}^{Prem,Res}}\right) - \text{P} - \text{reserve}^{initial}$$

Lemma: If $X \sim \mathcal{LN}$ with parameters m and s then $\text{VaR}_\alpha(X) = \exp\left(m + s\Phi^{-1}(\alpha)\right)$

continued on next page

Premium and reserve risk



DEVELOPMENT (cont'd)

Combined risk: $SCR_{NL}^{Prem,Res} = \text{VaR}_{99.5\%}\left(\widehat{R_{NL}^{Prem,Res}}\right) - \mu$

$$SCR_{NL}^{Prem,Res} = \mu\left(\frac{exp\left(\Phi^{-1}(99.5\%)\sqrt{\ln(1+c^{prem,res^2})}\right)}{\sqrt{1+c^{prem,res^2}}} - 1\right)$$

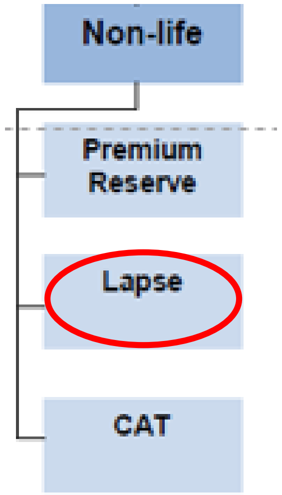
where:

μ is the expected value of the random variable of the premium and reserve risk (Art. 105(2))

$c^{prem,res}$ is the coefficient of variation of the random variable of premium and reserve risk

Simplification: $SCR_{NL}^{Prem,Res} \approx 3\mu \cdot c^{prem,res}$

Lapse risk



DEFINITION

Risk of massive cancelation of business that has a negative impact on reserves. This risk is only relevant if, for the determination of reserves, premium payments after the valuation year are being considered.

METHODOLOGY

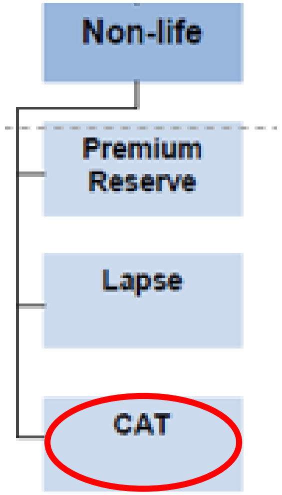
A scenario approach is followed:

$$SCR_{nl}^{c} = \max(E[\text{NAV}_{0\%}^{c}] - E[\text{NAV}_{50\%}^{c}], E[\text{NAV}_{0\%}^{c}] - E[\text{NAV}_{-50\%}^{c}])$$

For cash flow projections, the following assumptions are made:

- lapse rate is 50% higher than expected
- lapse rate is as expected
- lapse rate is 50% lower than expected

Catastrophe risk



DEFINITION

Risk of financial impact due to natural and man-made catastrophes.

METHODOLOGY

A factor approach is followed:

- Let P_i be the gross premium for each line of business $(i = 1, \dots, 13)$
- Let c_i be gross factors prescribed by EIOPA

$$SCR_{nl}^{cat} = \left[\left(\sqrt{\sum_{i \epsilon \{1,2,3,5\}} (c_i P_i)^2} + c_{11} P_{11} \right)^2 + \sum_{i \epsilon \{4,7,8,9,10,12\}} (c_i P_i)^2 + (c_6 P_6 + c_{13} P_{13})^2 \right]^{1/2}$$

Lines of business prescribed by EIOPA and c-factors

Events	Lines of business affected	Gross Factor c_t
Storm	Fire and property; Motor, other classes	175%
Flood	Fire and property; Motor, other classes	113%
Earthquake	Fire and property; Motor, other classes	120%
Hail	Motor, other classes	30%
Major fires, explosions	Fire and property	175%
Major MAT disaster	MAT	100%
Major motor vehicle liability disasters	Motor vehicle liability	40%
Major third party liability disaster	Third party liability	85%
Credit	Credit	139%
Miscellaneous	Miscellaneous	40%
NPL Property	NPL Property	250%
NPL MAT	NPL MAT	250%
NPL Casualty	NPL Casualty	250%

Combined non-life underwriting capital requirement

$$SCR_{nl} = \left[\left(SCR_{nl}^{pr}\right)^2 + (SCR_{nl}^{c})^2 + (SCR_{nl}^{cat})^2 + 2corrSCR_{nl}^{cat}\sqrt{\left(SCR_{nl}^{pr}\right)^2 + (SCR_{nl}^{c})^2}\right]^{1/2}$$

where $corrSCR_{nl}^{cat} = 0.25$ is the coefficient of correlation between the combined premium, reserve and lapse risks, and the catastrophe risk.

Illustrative example

GENERAL DATA

	Fire & property damage			Third-party liability			Business interruption		
	2017	2018	2019E	2017	2018	2019E	2017	2018	2019E
Gross written premium	10,000	12,000	-	1,000	1,200	-	2,000	2,500	-
Unearned premium reserve	1,000	1,150	1,200	100	135	140	275	350	400
Outstanding loss reserve	-	4,000	-	-	500	-	-	750	-
Quota-share cession	-	20%	-	-	25%	-	-	25%	-
Average claim	-	100	-	-	75	-	-	150	-
Coefficient of variation	-	0.50	-	-	0.60	-	-	0.75	-

RESERVE RUN-OFF PATTERNS

	year				
	1	2	3	4	5
Fire & property damage	0.588	0.353	0.059	-	-
Third-party liability	0.357	0.321	0.214	0.071	0.038
Business interruption	0.714	0.286	-	-	-

Model output

The **R** implementation of this model produced the following output:

```
> model_ouput
                                Amounts in USD
Premium and reserve risk:                21799
Lapse risk:                                  0
Catastrophe risk:                        10150
Diversification benefit:                 -5703
Combined risk:                           26246
Gross written premium:                   15700
Net earned premium (NEP):                12270
Combined risk as % of NEP:                 214
```

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